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Hamilton Formulation for Continuous Systems with Second Order Derivatives

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Abstract In this paper the Hamilton formulation for continuous systems with second order derivatives has been developed. We generalized the Hamilton formulation for continuous systems with second order derivatives and apply this new formulation to Podolsky generalized electrodynamics, comparing with the results obtained through Dirac's method.

Keywords Hamiltonian formalism · Constrained systems · Podolsky generalized electrodynamics

1 Introduction

Dirac's theory of constrained systems plays an important role in theoretical physics and is widely used in investigating theoretical models in a contemporary elementary particle physics [1–7]. In this formalism the constraints caused by the Hessian matrix singularity

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are added to the canonical Hamiltonian and then the consistency conditions are worked out, being possible to eliminate some degrees of freedom of the system. The presence of constraints requires care when applying Dirac's method, especially when first class-constraints arise, since the first class constraints are generators of gauge transformations which lead to the gauge freedom and one should impose gauge fixing condition for each first class constraints which are not always an easy task.

Systems with higher order Lagrangians have been studied with increasing interest because they appear in many relevant physical problems. The most popular examples being perhaps higher order regularization of quantum gauge field theories and so-called rigid strings [8, 9], rigid particles [10, 11], a relativistic particle with curvature and torsion in three dimensional space-time [12] and in the work of Podolsky [13] and Bopp [14], who independently proposed generalization of electrodynamics containing second order derivatives. Also the work of Green [15, 16], proposed a generalized meson-field theory. In addition of that, we have other examples, such as, the consistent of ultraviolet divergence in gauge invariant super symmetric theories [17, 18], or the effective Lagrangian in gauge theories [19, 20].

The treatments for theories with higher order derivatives has been first developed by Ostrogradski [21] and allow us to obtain the Euler-Lagrange equations and the Hamilton equations of motion. The path integral quantization of systems of higher order derivatives is studied [22]. Also, important applications in the field of the Hamilton-Jacobi formalism [23–30] were made in [31–33], including systems with higher order derivatives. Another work of the Hamilton-Jacobi formalism of systems with higher order derivatives is given in [34–41], where the action function is obtained for both constrained and unconstrained systems by solving the appropriate set of Hamilton-Jacobi partial differential equations, and used it to determine the solution of the equations of motion by using the WKB approximation.

By using a different approach some authors studied the Hamiltonian formulation of higher order dynamical systems using Dirac's approach to constrained dynamics, where Hamiltonian formulation of regular higher order Lagrangians is developed, and conventional description of such systems due to Ostrogradski is recovered [42, 43]. Also, a new development for systems with higher order derivatives and degenerate coordinate, was made in reference [44]. Very recently, a new development of systems with higher order fractional derivatives was made in reference [45], and the path integral quantization for both conservative and non conservative systems are recovered. Other treatments for systems with higher order derivatives are presented in reference [46], where the authors have analyzed systems with higher order derivatives, using the discrete variational principle to obtain the discrete Euler-Lagrange equations for higher-order Lagrangians and the corresponding discrete Hamiltonian.

Very recently, Muslih and El-Zalan has been developed new formulations to treat discrete systems with higher order Lagrangian. The path integral quantization is obtained as integration over the canonical coordinate without any need to integrate over the higher order derivatives [47].

In this paper, these formulations are generalized to be applicable for continuous systems with second order derivatives. The method is applied to Podolsky generalized electrodynamics.

The plan of the paper is as follows. We present in Sect. 2 an overview of the features of Lagrangian densities for free fields. In Sect. 3 we develop the Hamilton formulation for a general second order continuous system. In Sect. 4 an example of Podolsky electrodynamics for continuous systems with second order derivatives is solved using the new formalism [47]. Finally, Sect. 5 is devoted to the conclusions.

2 Lagrangian Densities for Free Fields

By analogy with discrete mechanics, we may canonically quantize a field $\phi_\rho(\mathbf{x},t)$ through the following prescription explained in the following steps.

We start by constructing a classical Lagrangian density $\mathcal{L}(\phi_\rho, \partial_\mu\phi_\rho)$ depending on generalized coordinates $\phi_\rho(\mathbf{x},t)$ and derivatives; the corresponding Lagrangian is given by

$$L = \int \mathcal{L}(\phi_\rho, \partial_\mu\phi_\rho) d^3x. \tag{1}$$

Integration of the Lagrangian values against time gives the action, a functional of path in configuration space as,

$$S = \int \mathcal{L}(\phi_\rho, \partial_\mu\phi_\rho) d^4x. \tag{2}$$

The evolution of the classical system is then obtained from the Euler-Lagrange equations of motion. These equations follow from the Hamilton's principle, the classical dynamical content of a system is prescribed by the requirement that the time integral of the Lagrangian (the classical action S) be an extremum.

$$\begin{aligned}\delta S &= \delta \int \mathcal{L}(\phi_\rho, \partial_\mu\phi_\rho) d^4x \\ &= \int \left(\frac{\partial\mathcal{L}}{\partial\phi_\rho}\delta\phi_\rho + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_\rho)}\delta(\partial_\mu\phi_\rho)\right) d^4x.\end{aligned} \tag{3}$$

In deriving (3), it was assumed that $\partial_\mu\phi_\rho = \frac{\partial\phi_\rho}{\partial x_\mu}$ are not independents of ϕ_ρ, so that $\delta(\partial_\mu\phi_\rho) = \partial_\mu(\delta\phi_\rho)$. This corresponds to the variational principle in configuration space.

Imposing $\delta S = 0$, the requirement that this variational equation be satisfied for arbitrary $\delta\phi_\rho$ leads to the Euler-Lagrange equations of motion

$$\frac{\partial\mathcal{L}}{\partial\phi_\rho} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_\rho)}\right) = 0. \tag{4}$$

We define the generalized momenta

$$\pi(\mathbf{x},t) = \frac{\partial\mathcal{L}}{\partial\dot{\phi}_\rho}, \tag{5}$$

where $\dot{\phi}_\rho \equiv \partial_0\phi_\rho$ and construct the classical Hamiltonian density $\mathcal{H}(\pi(\mathbf{x},t), \phi_\rho(\mathbf{x},t))$ by a Legendre transformation

$$\mathcal{H} = \pi\dot{\phi}_\rho - \mathcal{L}, \tag{6}$$

and integrate the Hamiltonian density to obtain the classical Hamiltonian

$$H = \int dx \mathcal{H}(\pi(\mathbf{x},t), \phi_\rho(\mathbf{x},t)). \tag{7}$$

Finally, we quantize the classical Hamiltonian by the standard operator replacements.

3 Hamiltonian Formulation of Continuous Systems of Second Order

The dynamics of a physical system is encoded in the Lagrangian, a function of the positions and velocities of all the degrees of freedom which comprise the system [6]. To extract the dynamics one considers paths in the configuration space. For a given path, one calculates the position and velocities at each time and also the value of the Lagrangian.

The continuous system with Lagrangian density denoted on the dynamical field variables, generalized coordinate ϕ_ρ and its derivatives upon second order—generalized velocities $\partial_\mu\phi_\rho$ defined as:

$$\mathcal{L} = \mathcal{L}(\phi_\rho, \partial_\mu\phi_\rho, \partial_\mu\partial_\mu\phi_\rho, t), \quad \mu = 0, i,\ i = 1, 2, 3. \tag{8}$$

So, the corresponding Lagrangian is given by

$$L = \int \mathcal{L}(\phi_\rho, \partial_\mu\phi_\rho, \partial_\mu\partial_\mu\phi_\rho, t) d^3x. \tag{9}$$

Integration of the Lagrangian values against time gives the action, a functional of path in configuration space as,

$$S = \int \mathcal{L}(\phi_\rho, \partial_\mu\phi_\rho, \partial_\mu\partial_\mu\phi_\rho, t) d^4x. \tag{10}$$

The evolution of the classical system is then obtained from the Euler-Lagrange equations of motion. So, we obtain the Euler-Lagrange equations of motion

$$\frac{\partial\mathcal{L}}{\partial\phi_\rho} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_\rho)}\right) + \partial_\mu\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\partial_\mu\phi_\rho)}\right) = 0. \tag{11}$$

The Hamiltonian formulations defined as

$$\mathcal{H} = p_1\dot{\phi}_\rho + p_2\ddot{\phi}_\rho - \mathcal{L}(\phi_\rho, \partial_\mu\phi_\rho, \partial_\mu\partial_\mu\phi_\rho, t). \tag{12}$$

Calculating the total differential of this Hamiltonian we obtain

$$\begin{aligned} d\mathcal{H} = {} & p_1 d\dot{\phi}_\rho + \dot{\phi}_\rho dp_1 + p_2 d\ddot{\phi}_\rho + \ddot{\phi}_\rho dp_2 \\ & - \frac{\partial\mathcal{L}}{\partial\phi_\rho} d\phi_\rho - \frac{\partial\mathcal{L}}{\partial(\dot{\phi}_\rho)} d(\dot{\phi}_\rho) - \frac{\partial\mathcal{L}}{\partial(\partial_i\phi_\rho)} d(\partial_i\phi_\rho) \\ & - \frac{\partial\mathcal{L}}{\partial(\ddot{\phi}_\rho)} d(\ddot{\phi}_\rho) - \frac{\partial\mathcal{L}}{\partial(\partial_i\partial_i\phi_\rho)} d(\partial_i\partial_i\phi_\rho) - \frac{\partial\mathcal{L}}{\partial t} dt. \end{aligned} \tag{13}$$

The generalized momenta p_1 and p_2 corresponding to $\dot{\phi}_\rho = \partial_0\phi_\rho$ and $\ddot{\phi}_\rho = \partial_0\partial_0\phi_\rho$ can be defined as follows

$$p_1 = \frac{\partial\mathcal{L}}{\partial(\dot{\phi}_\rho)}, \tag{14}$$

$$p_2 = \frac{\partial\mathcal{L}}{\partial(\ddot{\phi}_\rho)}. \tag{15}$$

Substituting the value of momenta (14) and (15) into (13) we have:

$$d\mathcal{H} = \dot{\phi}_\rho dp_1 + \ddot{\phi}_\rho dp_2 - \frac{\partial \mathcal{L}}{\partial \phi_\rho} d\phi_\rho - \frac{\partial \mathcal{L}}{\partial(\partial_i \phi_\rho)} d(\partial_i \phi_\rho) - \frac{\partial \mathcal{L}}{\partial(\partial_i \partial_i \phi_\rho)} d(\partial_i \partial_i \phi_\rho) - \frac{\partial \mathcal{L}}{\partial t} dt. \tag{16}$$

Making use of the Euler-Lagrange equation (11), we get

$$d\mathcal{H} = \dot{\phi}_\rho dp_1 + \ddot{\phi}_\rho dp_2 + \left[-\dot{p}_1 + \ddot{p}_2 - \partial_i \left(\frac{\partial \mathcal{L}}{\partial(\partial_i \phi_\rho)}\right) + \partial_i \partial_i \left(\frac{\partial \mathcal{L}}{\partial(\partial_i \partial_i \phi_\rho)}\right)\right] d\phi_\rho - \frac{\partial \mathcal{L}}{\partial(\partial_i \phi_\rho)} d(\partial_i \phi_\rho) - \frac{\partial \mathcal{L}}{\partial(\partial_i \partial_i \phi_\rho)} d(\partial_i \partial_i \phi_\rho) - \frac{\partial \mathcal{L}}{\partial t} dt. \tag{17}$$

This means that, the Hamiltonian is a function of the form

$$\mathcal{H} = \mathcal{H}(\phi_\rho, p_1, p_2, \partial_i \phi_\rho, \partial_i \partial_i \phi_\rho, t), \tag{18}$$

and the total differential of this function reads as:

$$d\mathcal{H} = \frac{\partial \mathcal{H}}{\partial \phi_\rho} d\phi_\rho + \frac{\partial \mathcal{H}}{\partial p_1} dp_1 + \frac{\partial \mathcal{H}}{\partial p_2} dp_2 + \frac{\partial \mathcal{H}}{\partial(\partial_i \phi_\rho)} d(\partial_i \phi_\rho) + \frac{\partial \mathcal{H}}{\partial(\partial_i \partial_i \phi_\rho)} d(\partial_i \partial_i \phi_\rho) + \frac{\partial \mathcal{H}}{\partial t} dt. \tag{19}$$

Comparing (17) and (19), we get the following Hamilton's equations of motion

$$\dot{\phi}_\rho = \frac{\partial \mathcal{H}}{\partial p_1}, \qquad \ddot{\phi}_\rho = \frac{\partial \mathcal{H}}{\partial p_2}, \tag{20}$$

$$\frac{\partial \mathcal{H}}{\partial(\partial_i \phi_\rho)} = -\frac{\partial \mathcal{L}}{\partial(\partial_i \phi_\rho)}, \qquad \frac{\partial \mathcal{H}}{\partial(\partial_i \partial_i \phi_\rho)} = -\frac{\partial \mathcal{L}}{\partial(\partial_i \partial_i \phi_\rho)}, \tag{21}$$

$$\frac{\partial \mathcal{H}}{\partial t} = -\frac{\partial \mathcal{L}}{\partial t}, \tag{22}$$

$$\frac{\partial \mathcal{H}}{\partial \phi_\rho} = -\dot{p}_1 + \ddot{p}_2 - \partial_i \left(\frac{\partial \mathcal{L}}{\partial(\partial_i \phi_\rho)}\right) + \partial_i \partial_i \left(\frac{\partial \mathcal{L}}{\partial(\partial_i \partial_i \phi_\rho)}\right). \tag{23}$$

4 An Example

As an example, let us consider the case of Podolsky electrodynamics which is based on the following Lagrangian [25, 48]

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + a^2 \partial_\lambda F^{\alpha\lambda} \partial_\rho F^\rho_\alpha, \tag{24}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

So, we can write the Lagrangian (24) as:

$$\mathcal{L} = -\frac{1}{2} F_{0i} F^{0i} + a^2 [(\partial_i F^{0i})^2 - (\partial_0 F^{0i})^2]. \tag{25}$$

An analysis of the Hamiltonian formalism for this theory carried out in [25, 48] and we compare some of the result presented there with the formalism developed here.

The momenta $p_1^{(1)}$, $p_1^{(2)}$, $p_2^{(1)}$ and $p_2^{(2)}$ are given as

$$p_1^{(1)} = \frac{\partial \mathcal{L}}{\partial(\partial_0 A_0)} = 0, \tag{26}$$

$$p_1^{(2)} = \frac{\partial \mathcal{L}}{\partial(\partial_0 A_i)} = F_i^0 = \partial_0 A_i - \partial_i A_0, \tag{27}$$

$$p_2^{(1)} = \frac{\partial \mathcal{L}}{\partial(\partial_0 \partial_0 A_0)} = 0, \tag{28}$$

$$p_2^{(2)} = \frac{\partial \mathcal{L}}{\partial(\partial_0 \partial_0 A_i)} = -2a^2 \partial_0 F_i^0 = -2a^2(\partial_0 \partial_0 A_i - \partial_0 \partial_i A_0). \tag{29}$$

The primary constraints are:

$$\Phi_1 = p_1^{(1)} = 0, \tag{30}$$

$$\Phi_2 = p_2^{(1)} = 0. \tag{31}$$

Making use of (12) the Hamiltonian becomes

$$\begin{aligned}\mathcal{H} &= p_1^{(1)} \partial_0 A_0 + p_1^{(2)} \partial_0 A_i + p_2^{(1)} \partial_0^2 A_0 + p_2^{(2)} \partial_0^2 A_i - \mathcal{L}(\phi_\rho, \partial_\mu \phi_\rho, \partial_\mu \partial_\mu \phi_\rho, t) \\ &= \frac{1}{2}(p_1^{(2)})^2 + p_1^{(2)} \partial_i A_0 - a^2[\partial_i p_1^{(2)} \partial_k p_1^{(2)} - \partial_i F^{ij} \partial_k F^{kj}] \\ &\quad + \frac{1}{4a^2}(p_2^{(2)})^2 + \frac{p_2^{(2)}}{2}[2\partial_0 \partial_i A_0 + \partial_k F^{kj} - \partial_i F^{ij}] + \frac{1}{4} F_{ik} F^{ik}\end{aligned} \tag{32}$$

and the Hamilton's equations of motions are

$$\partial_0 A_i = \frac{\partial \mathcal{H}}{\partial p_1^{(2)}} = p_1^{(2)} + \partial_i A_0, \tag{33}$$

$$\partial_0 \partial_0 A_i = \frac{\partial \mathcal{L}}{\partial p_2^{(2)}} = \frac{1}{2a^2} p_2^{(2)} + \partial_0 \partial_i A_0, \tag{34}$$

$$\begin{aligned}\frac{\partial \mathcal{H}}{\partial A_0} &= -\dot{p}_1^{(1)} + \ddot{p}_2^{(1)} - \partial_i \left(\frac{\partial \mathcal{L}}{\partial(\partial_i A_0)} \right) + \partial_i \partial_i \left(\frac{\partial \mathcal{L}}{\partial(\partial_i \partial_i A_0)} \right) \\ &\quad \partial_i[-\partial_0 A_i + \partial_i A_0] + \partial_i \partial_i [2a^2 \partial_i \partial_i A_0 - 2a^2 \partial_i \partial_0 A_i] = 0\end{aligned} \tag{35}$$

$$\begin{aligned}\frac{\partial \mathcal{H}}{\partial A_i} &= -\dot{p}_1^{(2)} + \ddot{p}_2^{(2)} - \partial_i \left(\frac{\partial \mathcal{L}}{\partial(\partial_i A_i)} \right) + \partial_i \partial_i \left(\frac{\partial \mathcal{L}}{\partial(\partial_i \partial_i A_i)} \right) \\ &\quad - \partial_0 F_i^0 - 2a^2 \partial_0^3 F_i^0 = 0.\end{aligned} \tag{36}$$

Equation (35) and (36) respectively give:

$$[1 + 2a^2 \partial_i^2] \partial_i F_i^0 = 0, \tag{37}$$

$$[1 + 2a^2 \partial_0^2] \partial_0 F_i^0 = 0, \tag{38}$$

or

$$[1 + 2a^2(\partial_0^2 - \partial_i^2)][\partial_0 + \partial_i]F_i^0 = 0. \tag{39}$$

The result obtained in (39) is the same as obtained in [25, 48].

5 Conclusions

Finding new formalisms in the field of constrained system lead us to the possibility to analyze a given problem by several methods and choose the appropriate one for a given problem. In this paper we have studied the Hamiltonian formulation for continuous systems with second order derivatives and we presented the Hamilton equations. We have analyzed in details the system of Podolsky electrodynamics for continuous systems with second order derivatives using the new formalism introduced [47]. The obtained results were found in complete agreement with those derived from the Lagrangian method by using Dirac's method.

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